GRAPH RECONSTRUCTION FROM RANDOM SUBGRAPHS

Andrew McGregor & Rik Sengupta

University of Massachusetts Amherst

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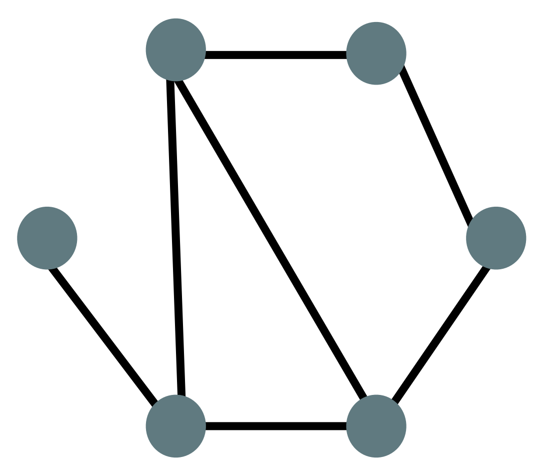

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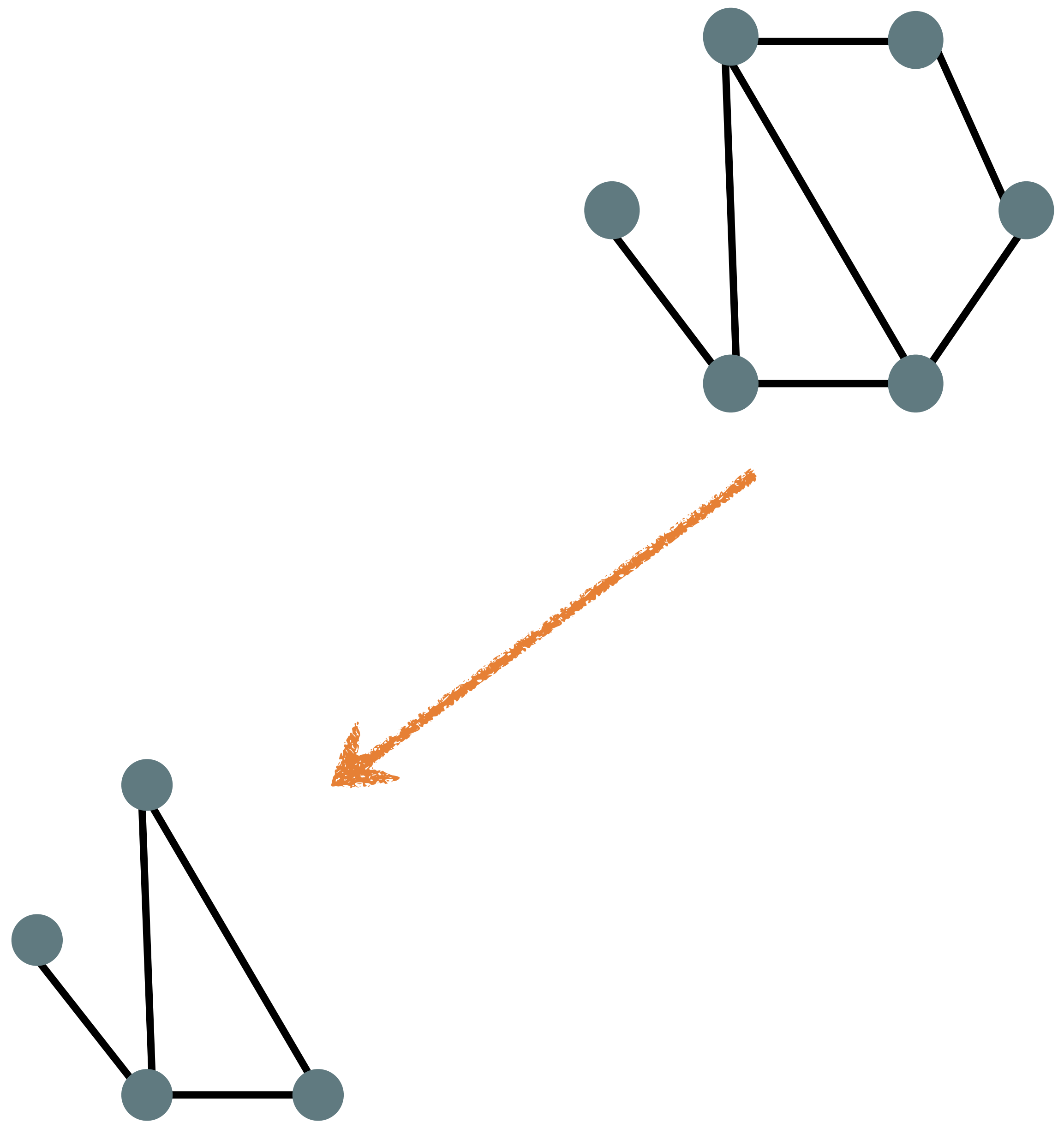

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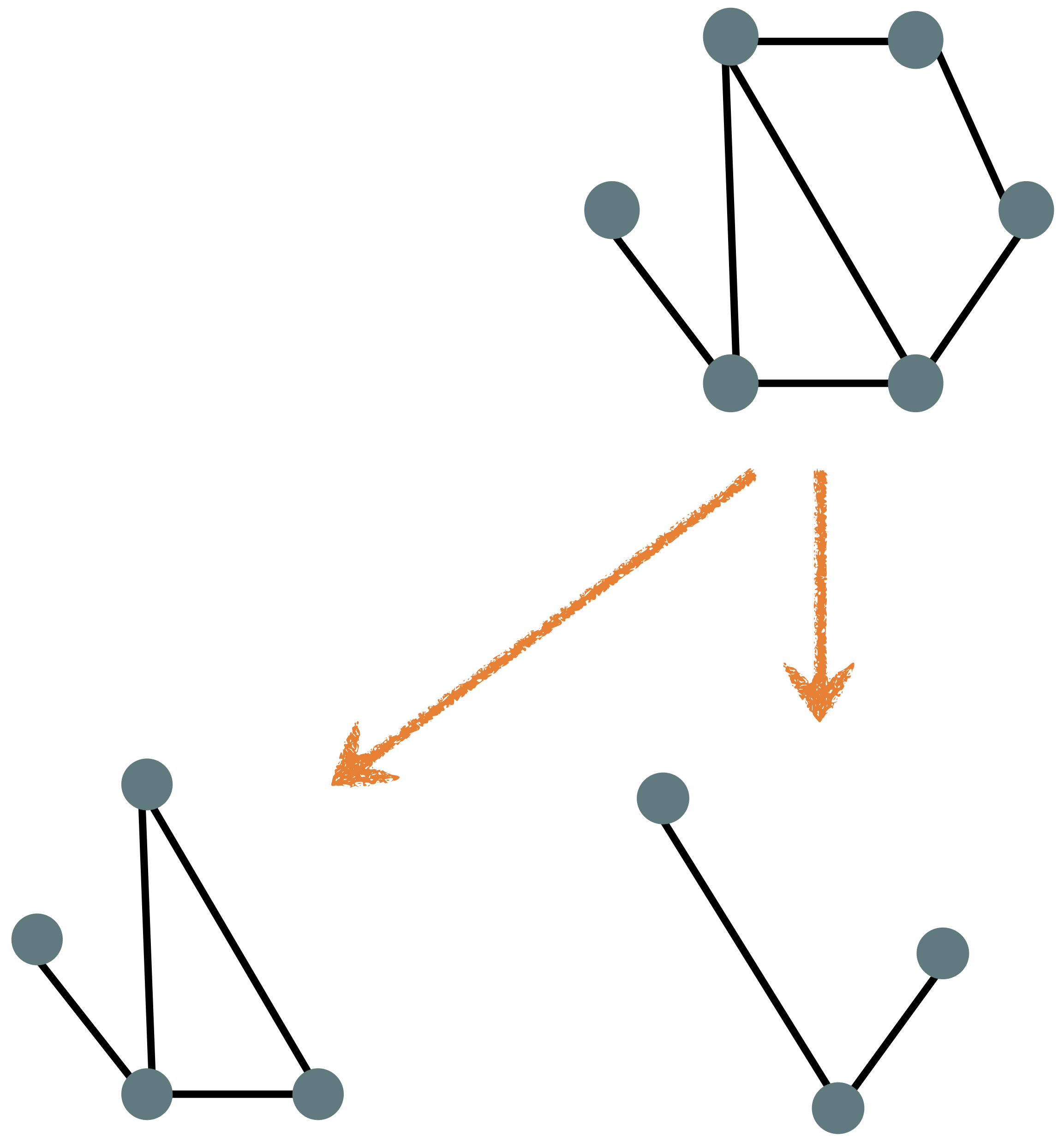

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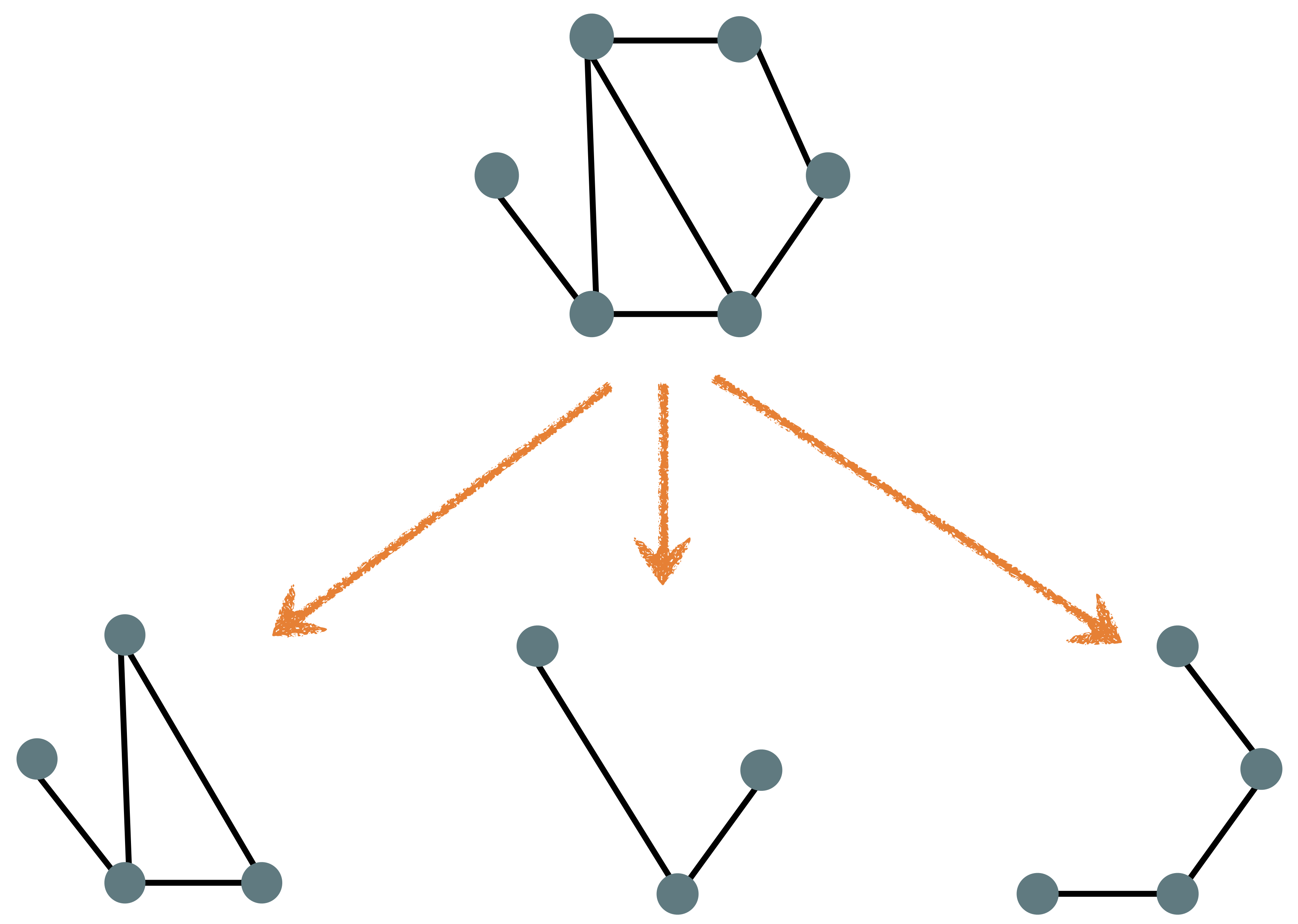

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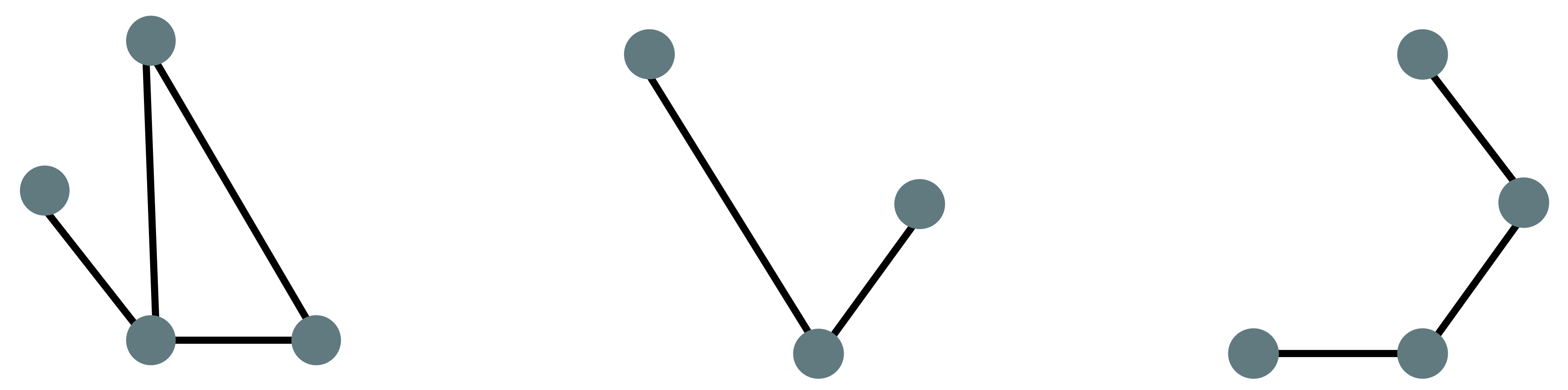

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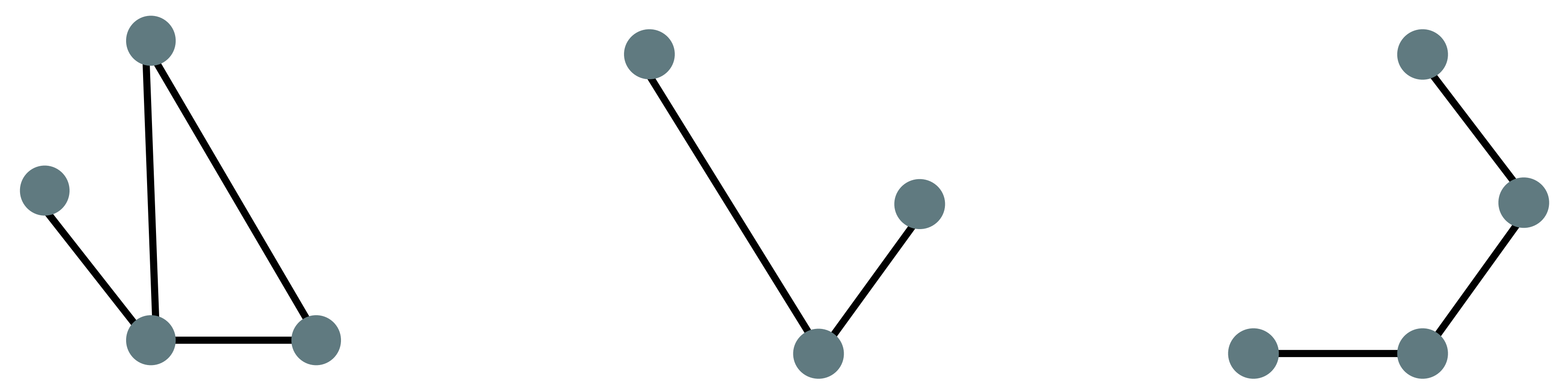

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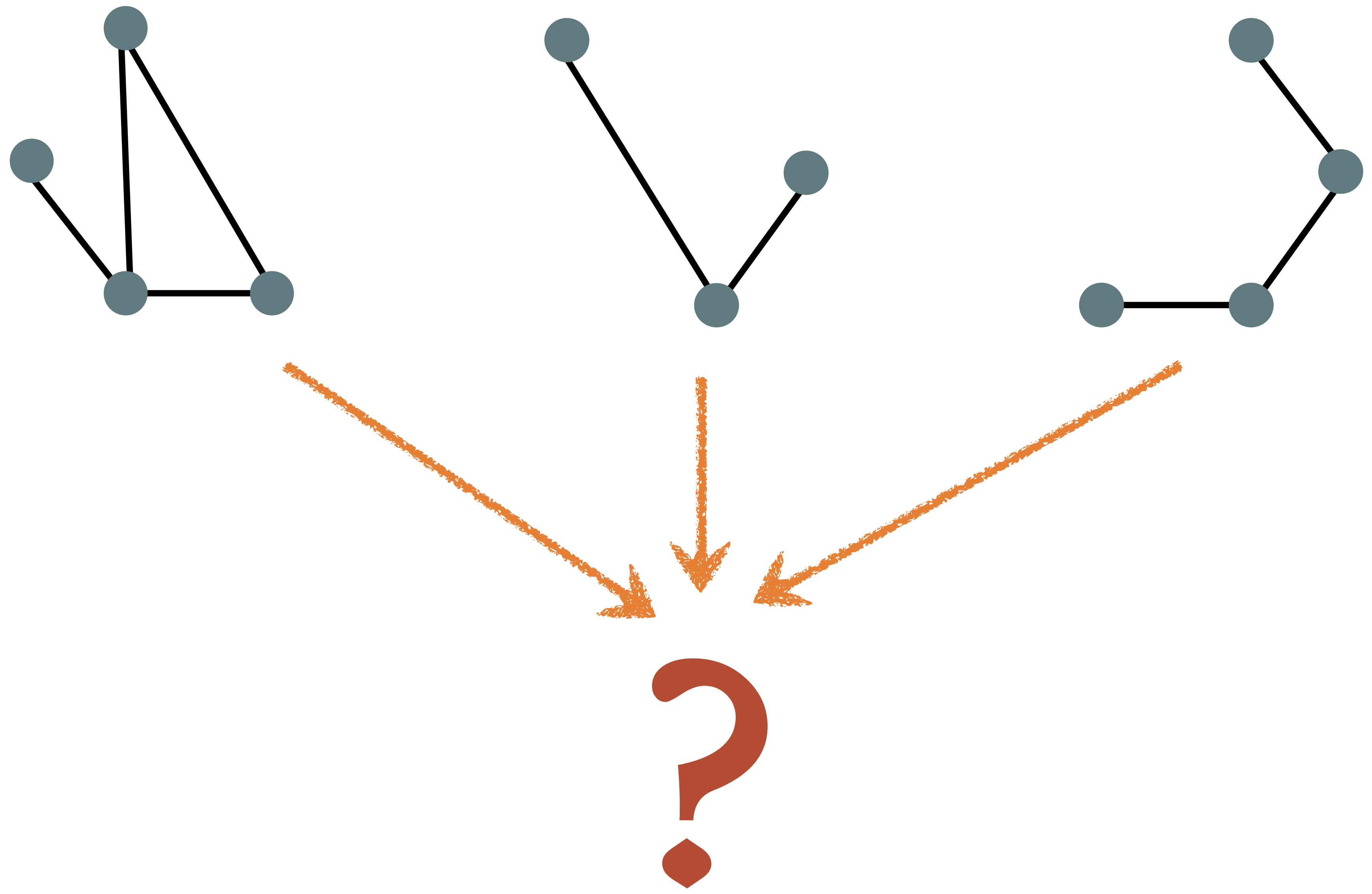

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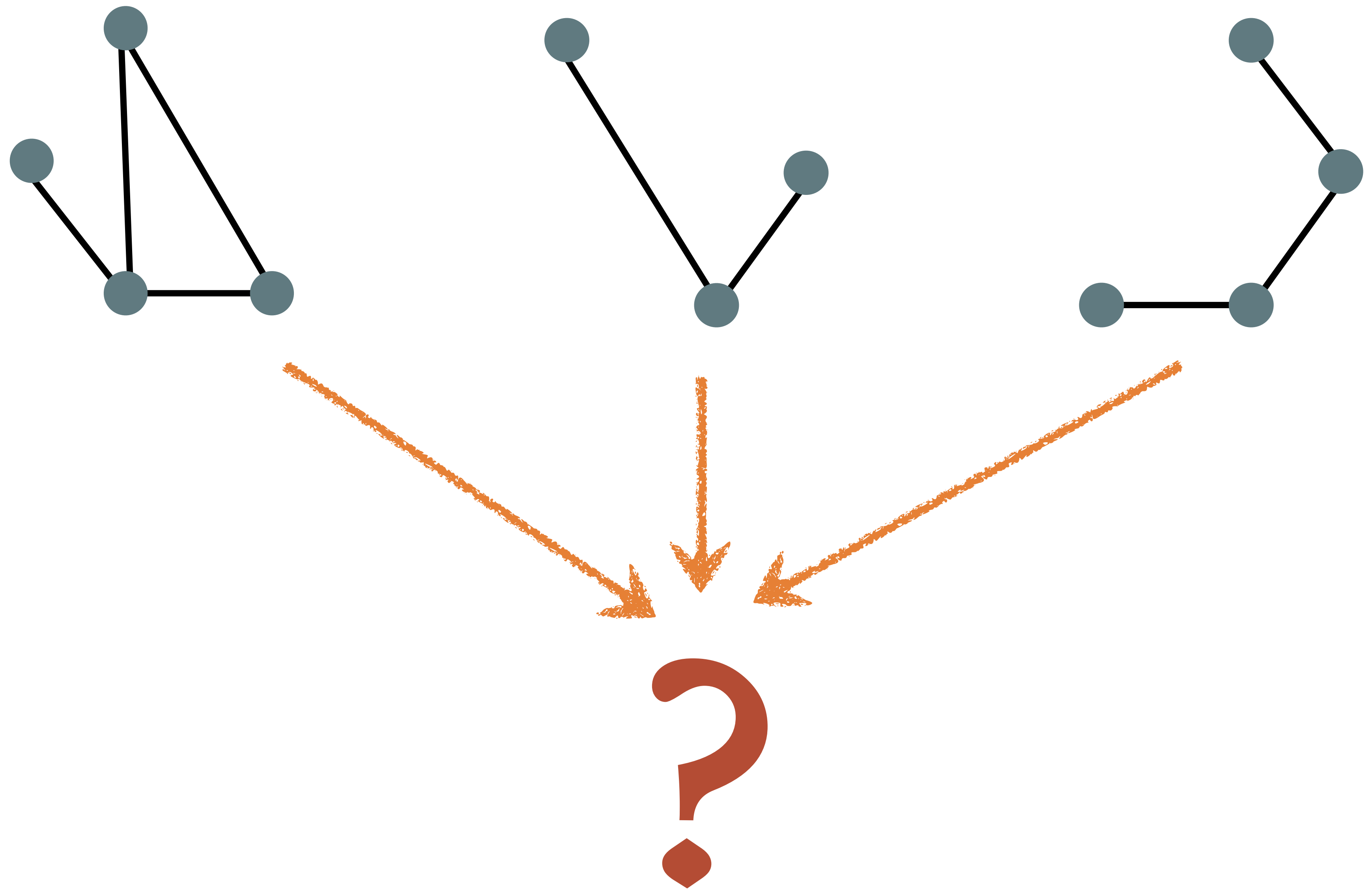
- Of course, this is easy if the nodes were *labeled*, but we are assuming we don't know which nodes were retained.



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  - For sparse graphs, $2^{O\left(n^{1/3}\right)}$ traces suffice.

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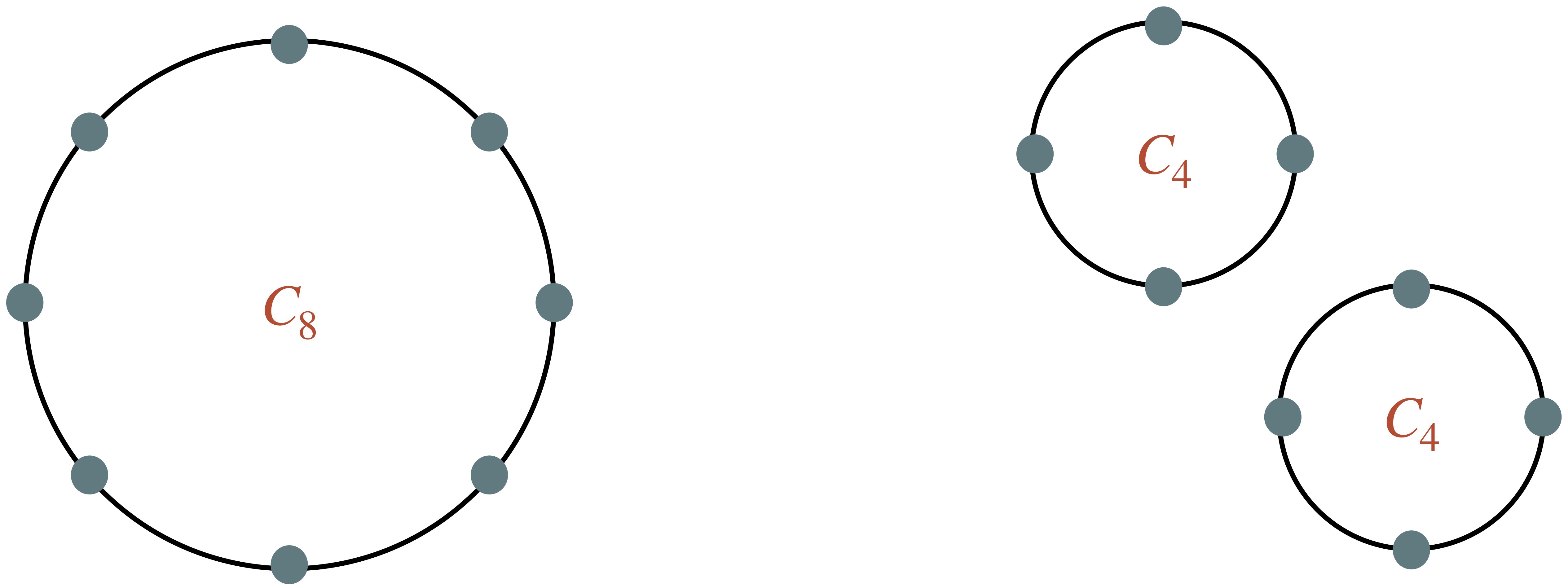

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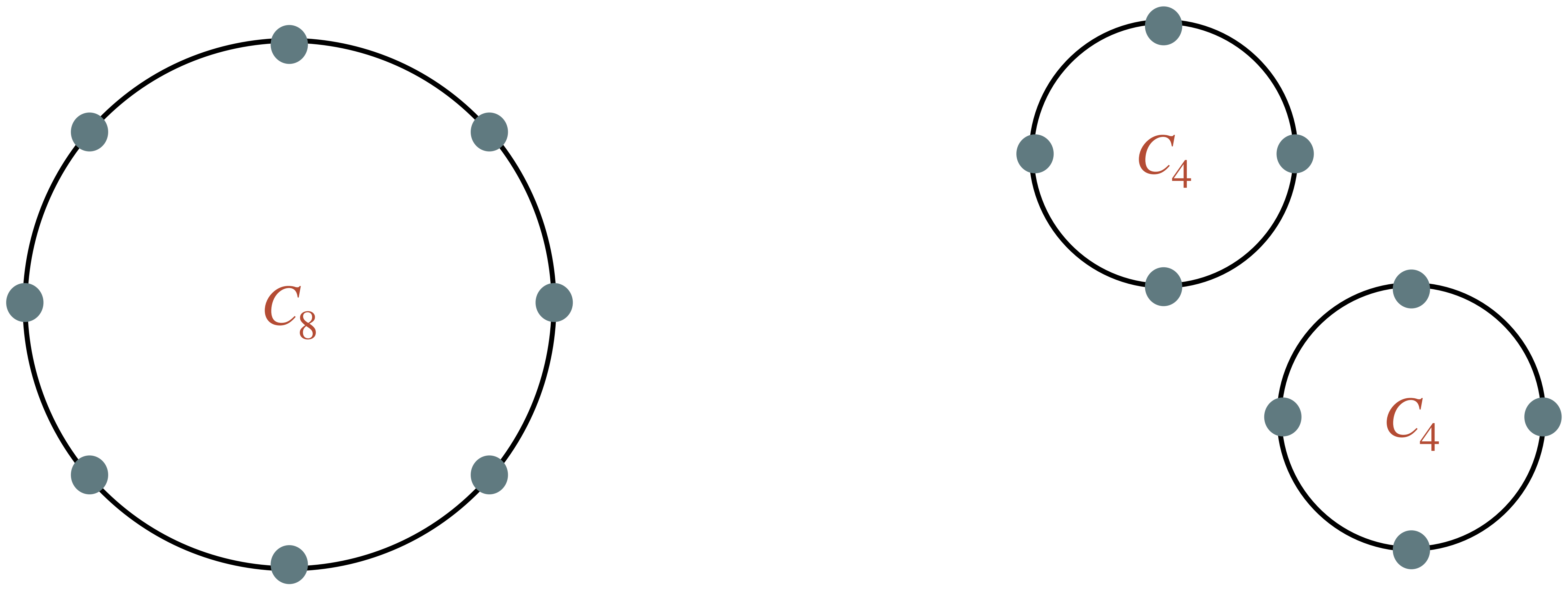

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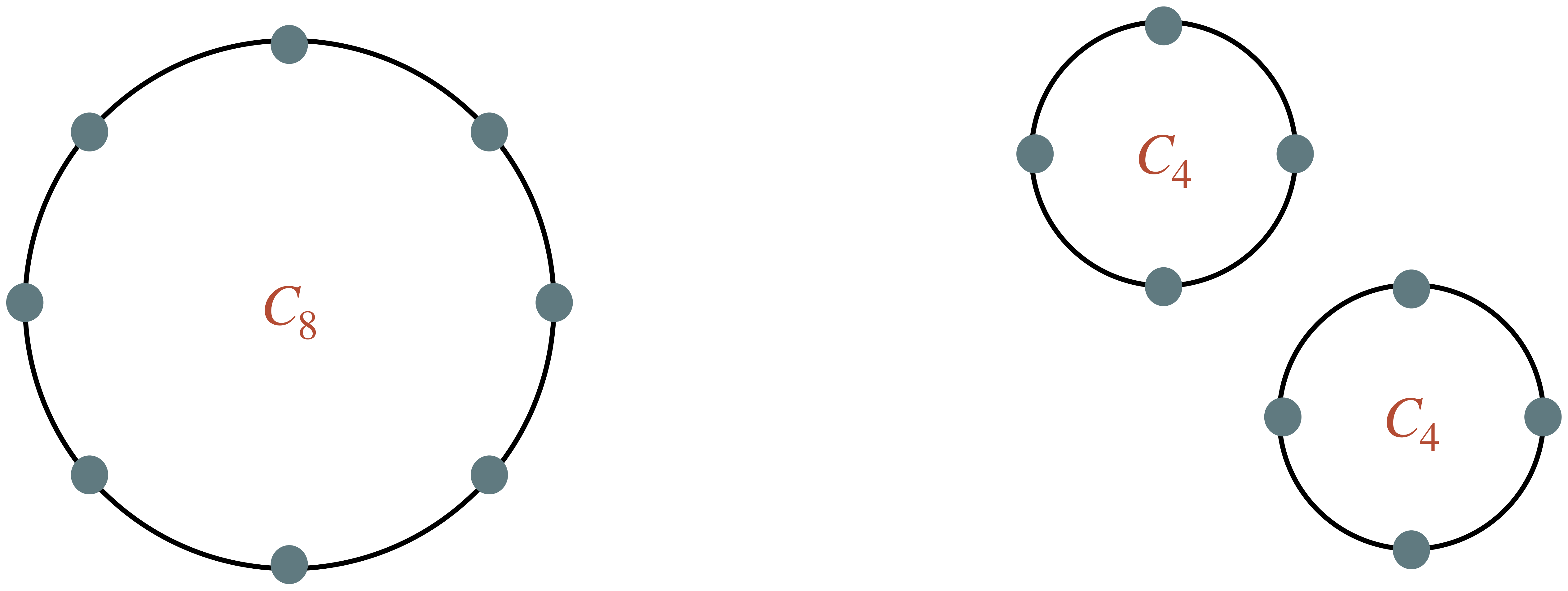

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Can show the distribution of subgraphs to be virtually identical, to bound error.

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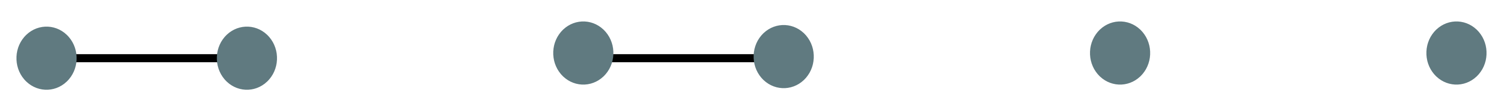

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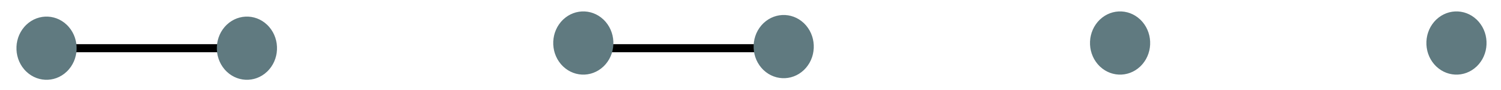

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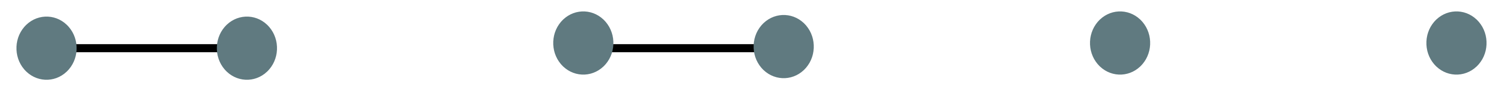
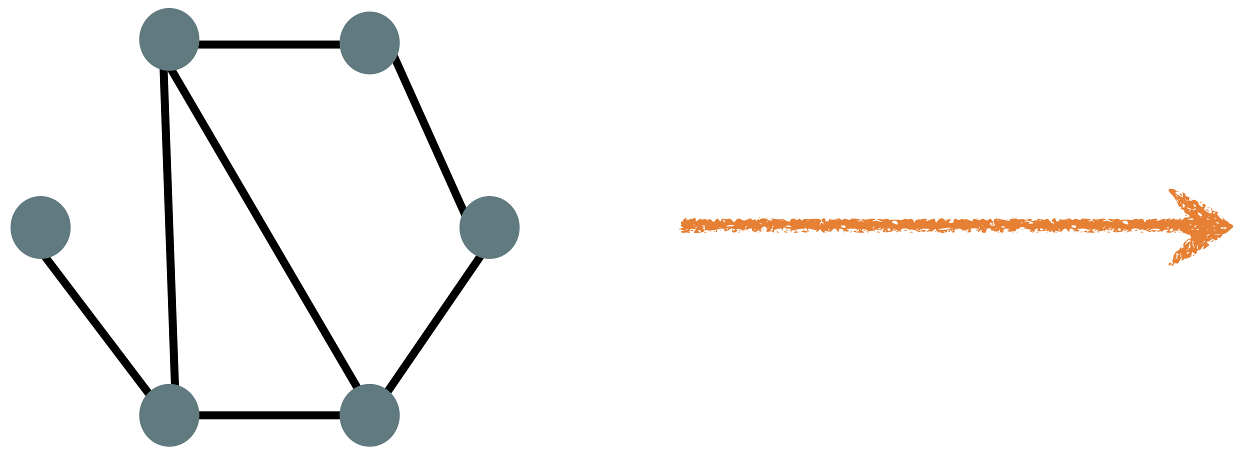


- **Degree Distribution**
  - Theorem. For any graph, we can recover the degree distribution in $\exp(O(n^{1/3}))$ traces whp.



$(3,3,3,2,2,1)$

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- Main Idea. Reconstruction would be easy if the nodes came with labels, so we try to identify common substructures to determine a *consistent* labeling of vertices across traces.
- Intermediate Question. How large a common substructure do we need to be sure that they correspond to the same nodes of the original graph?

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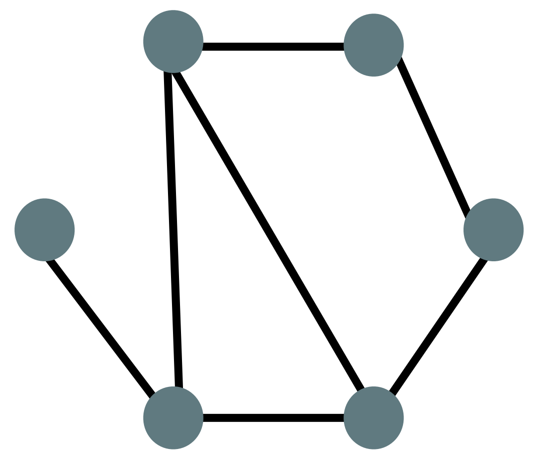

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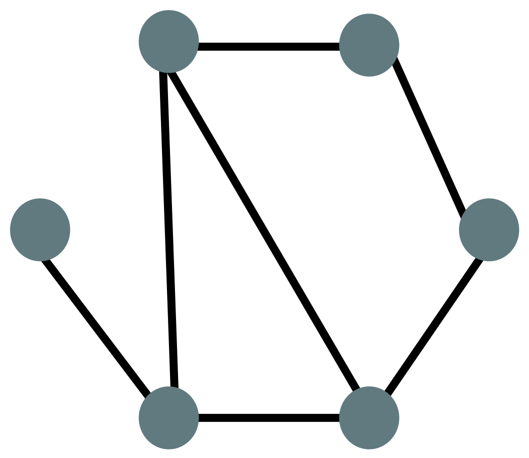

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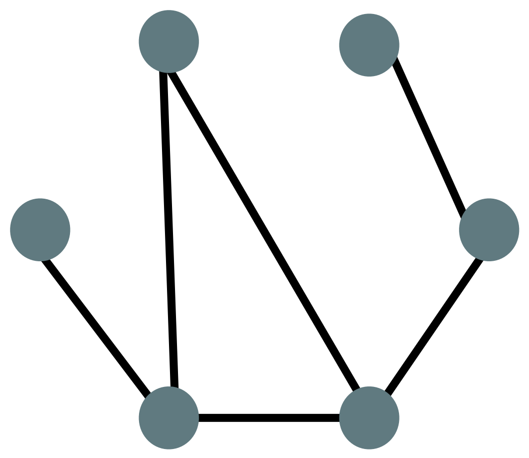

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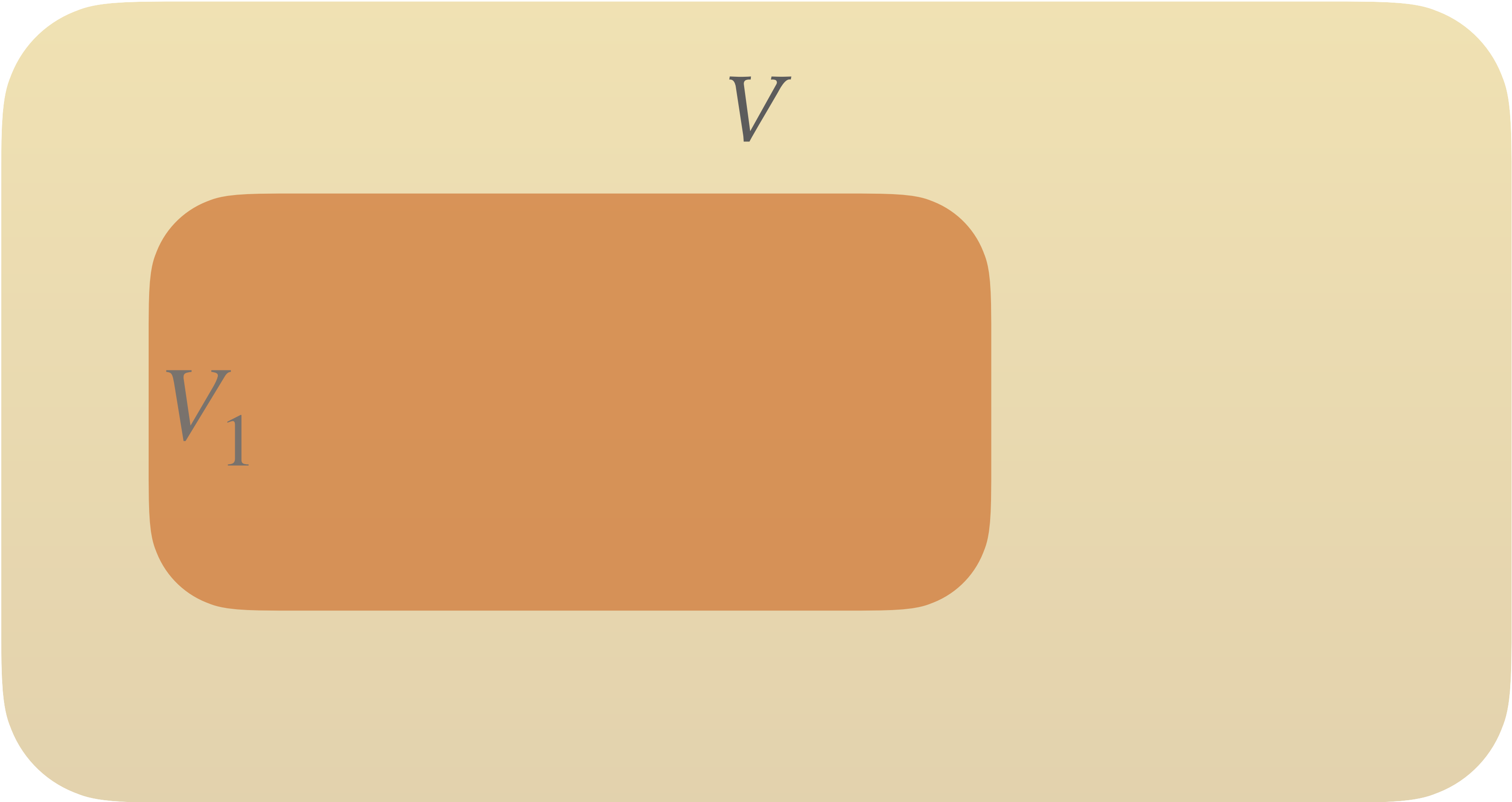

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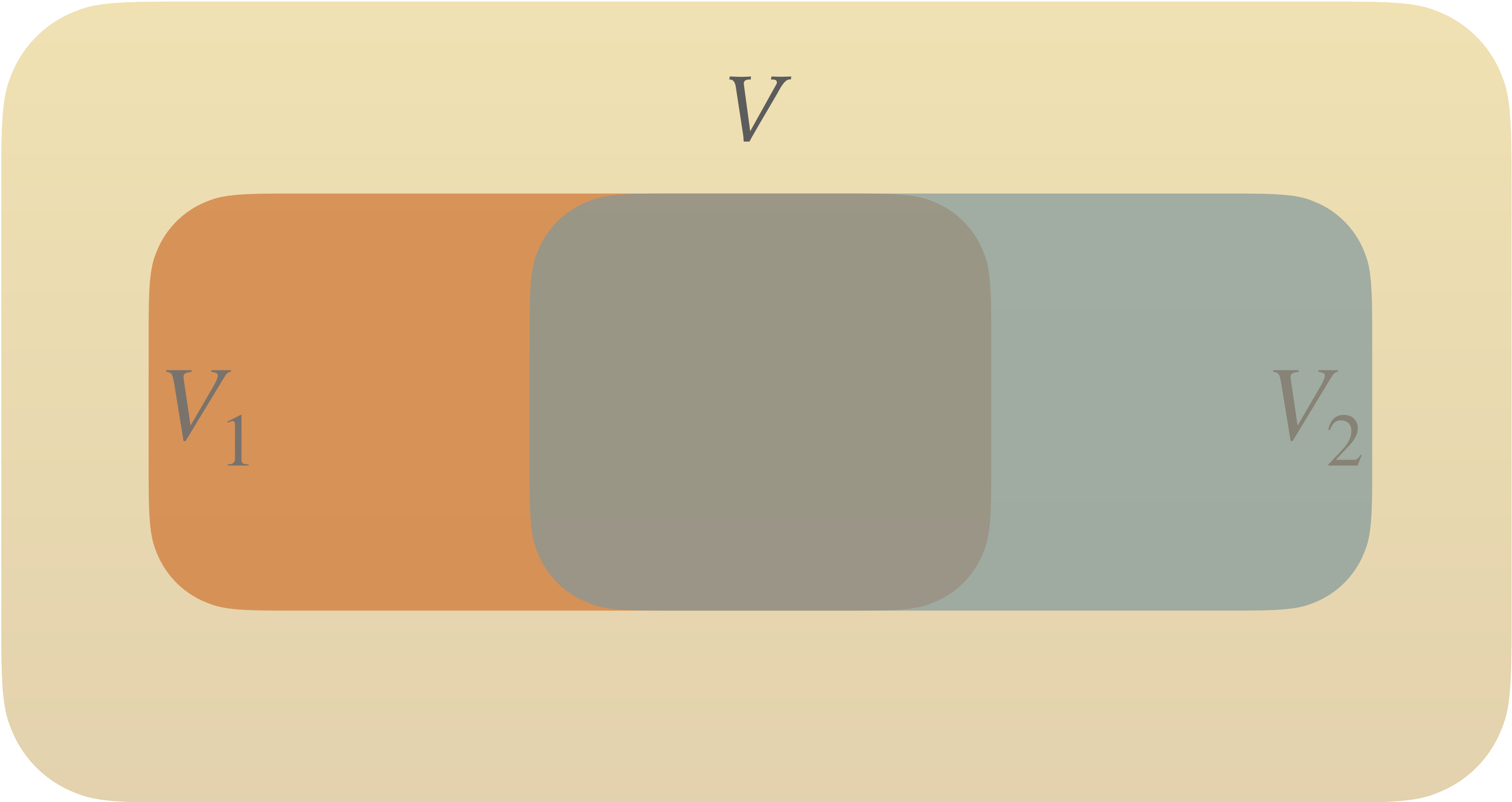

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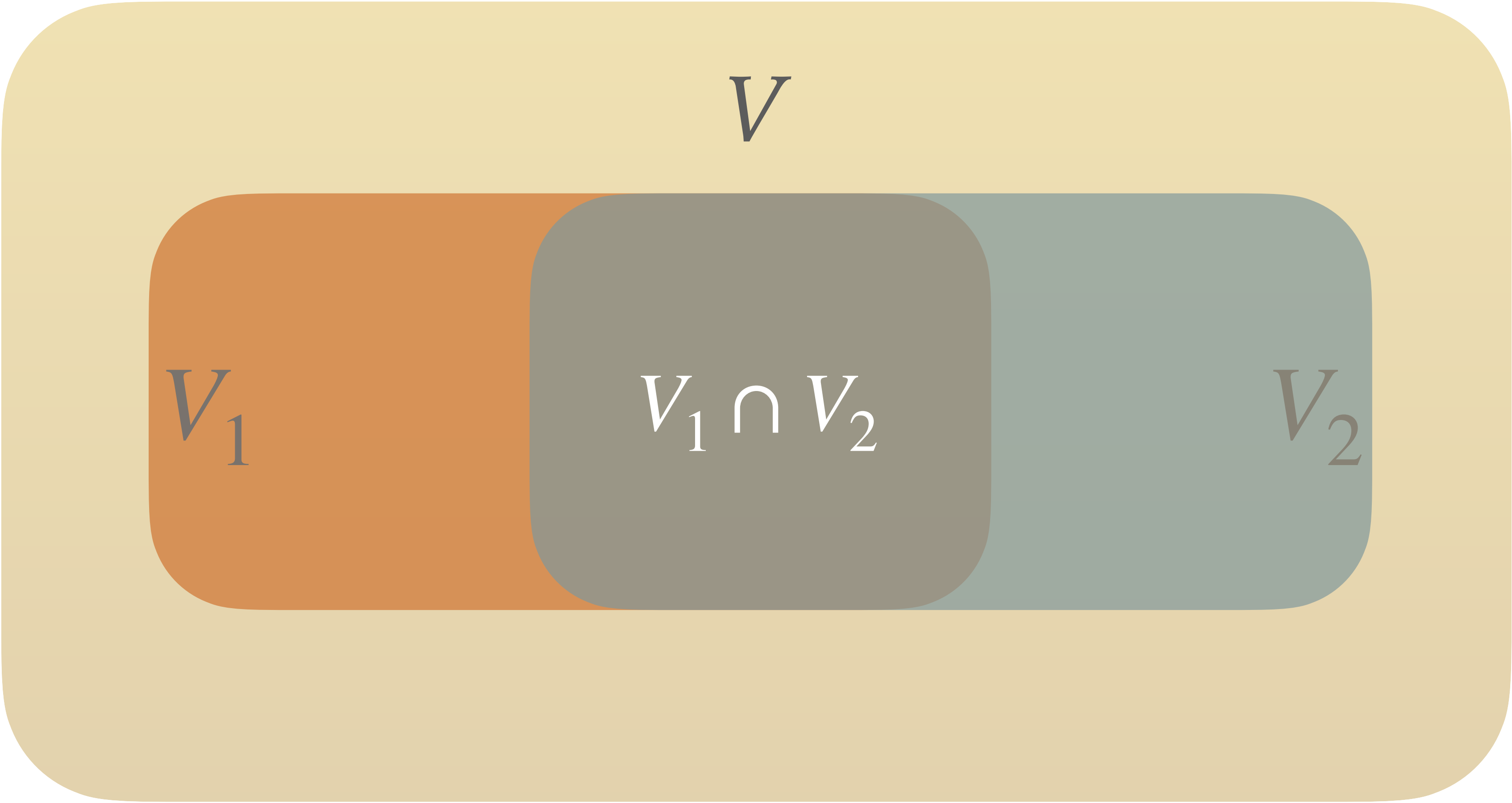

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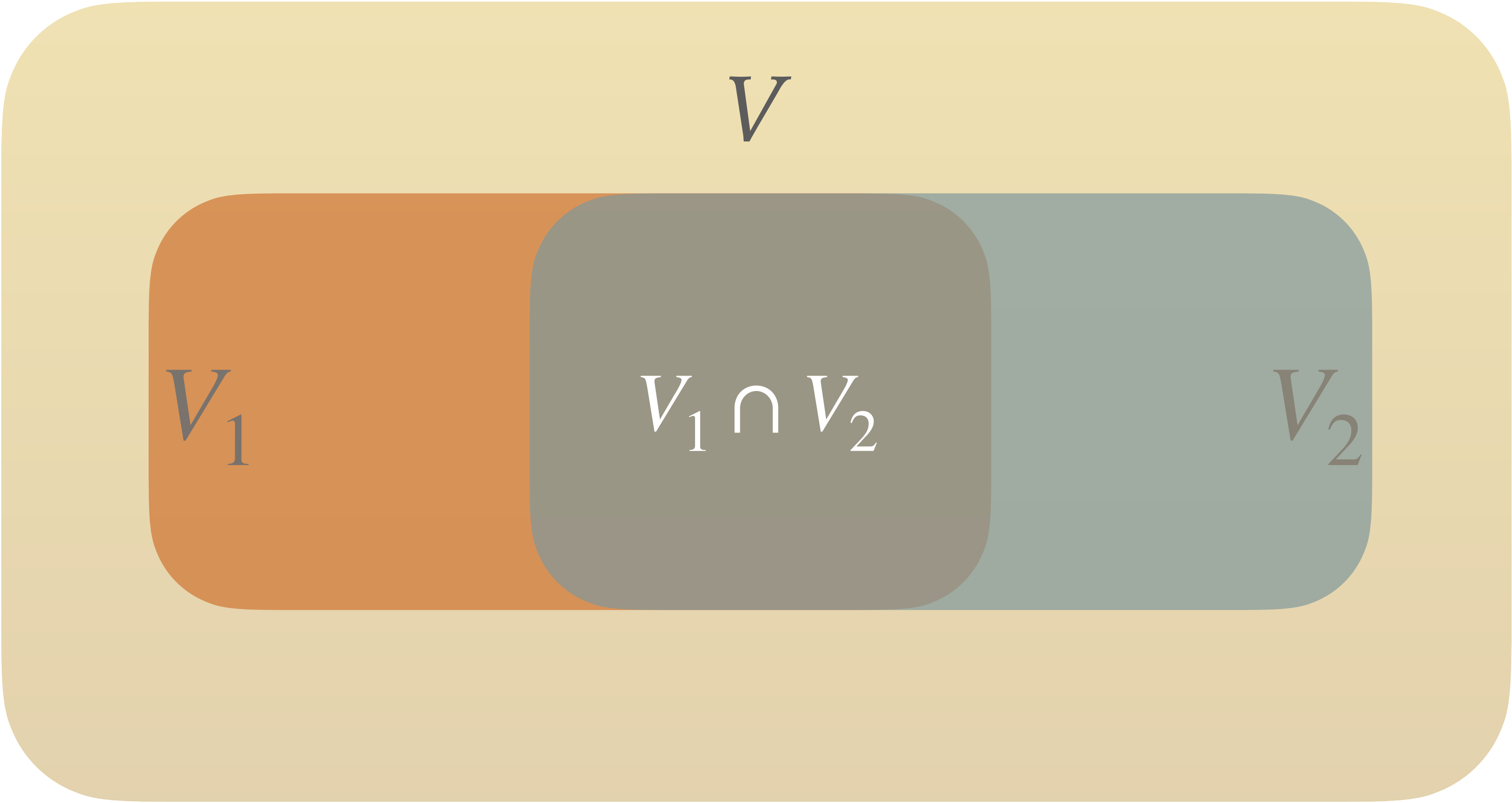

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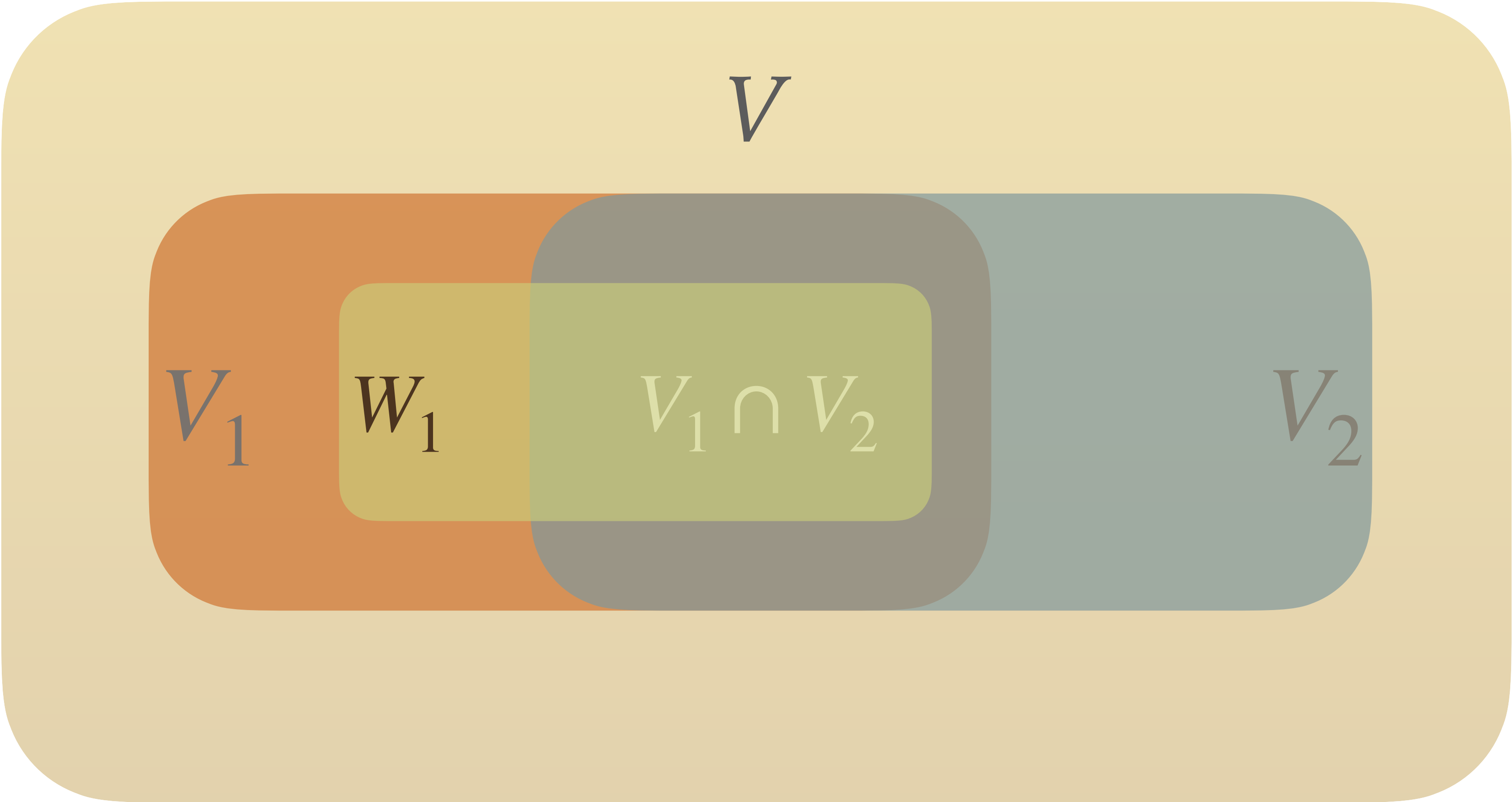

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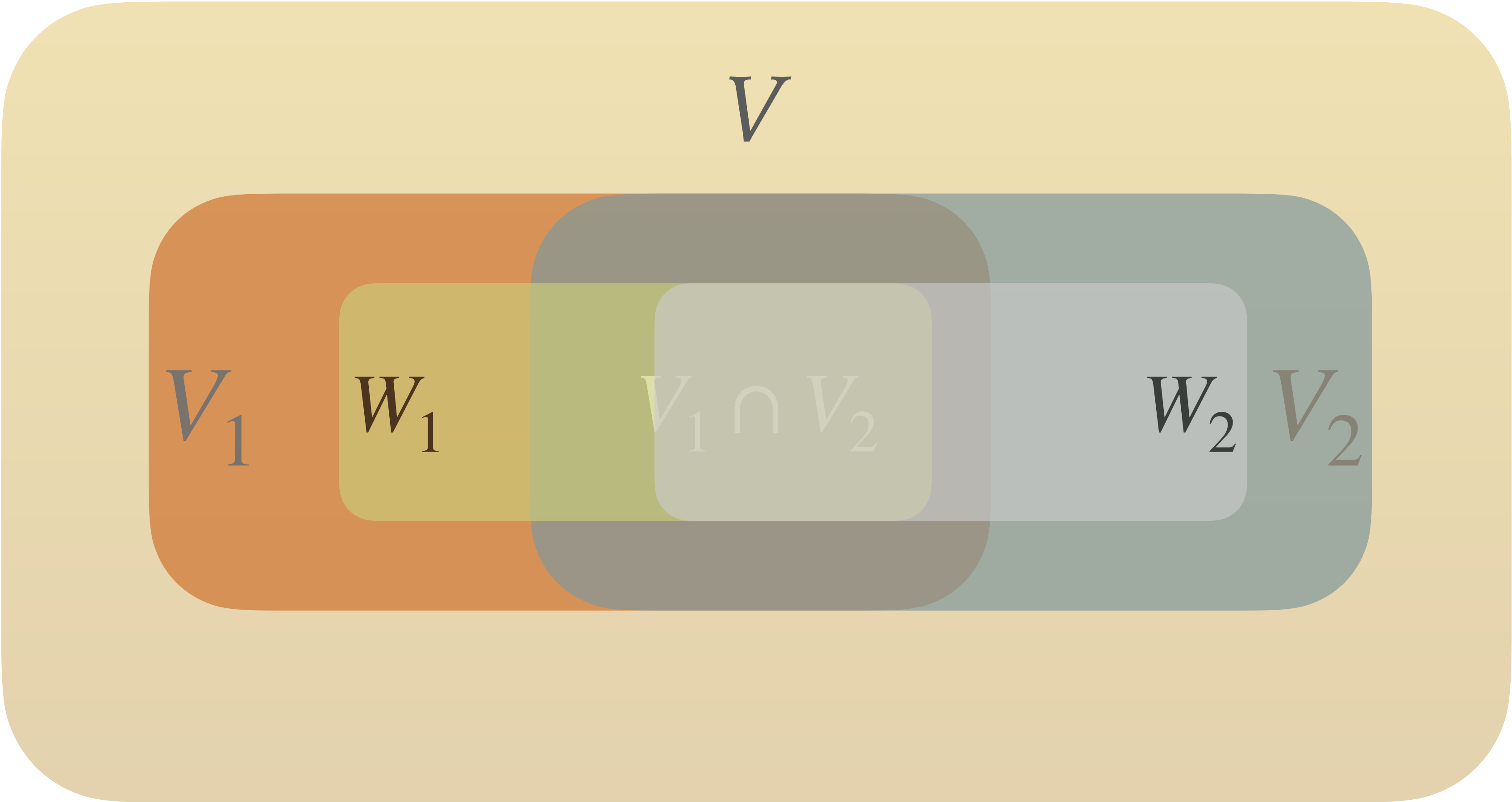

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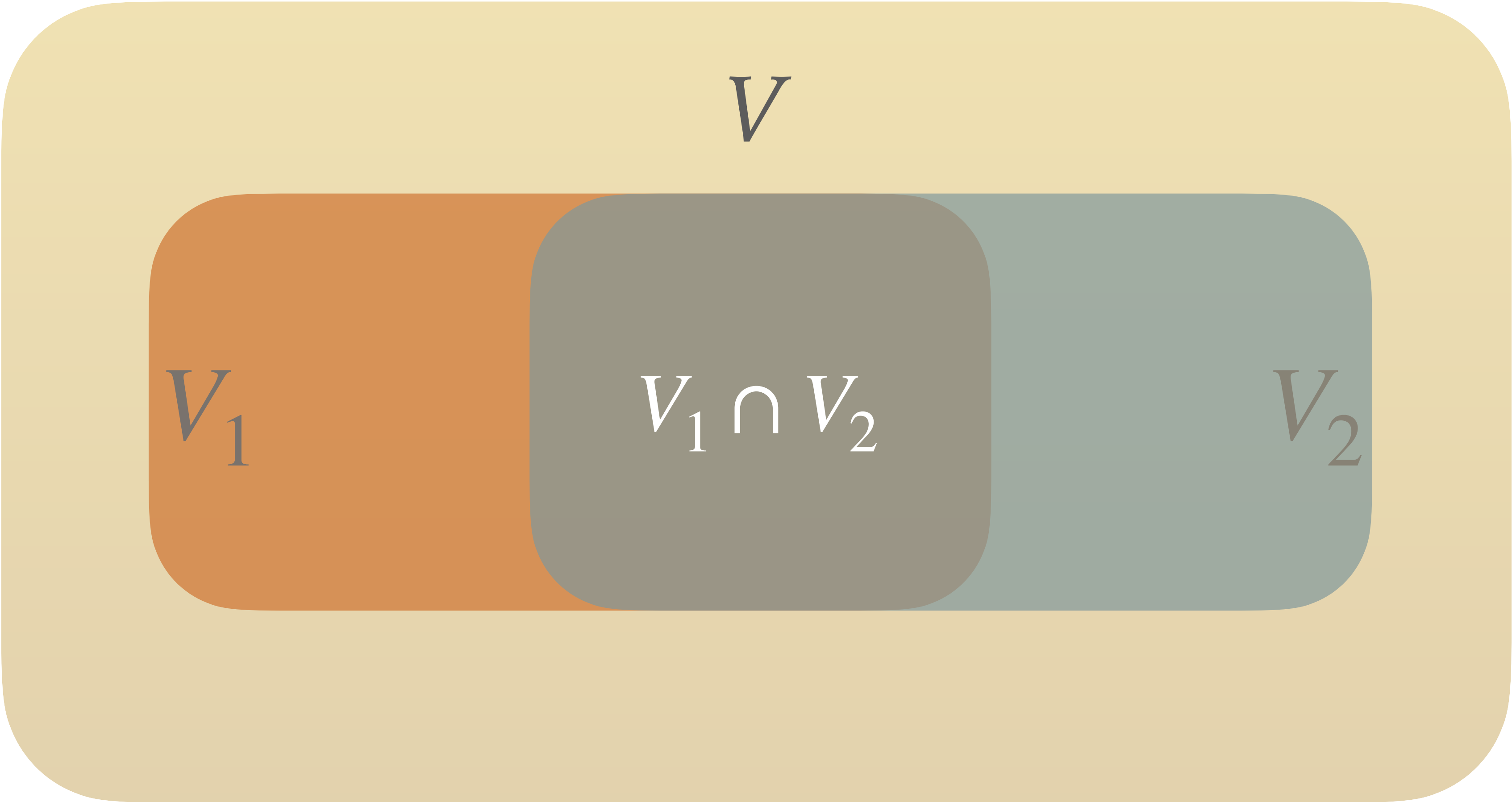

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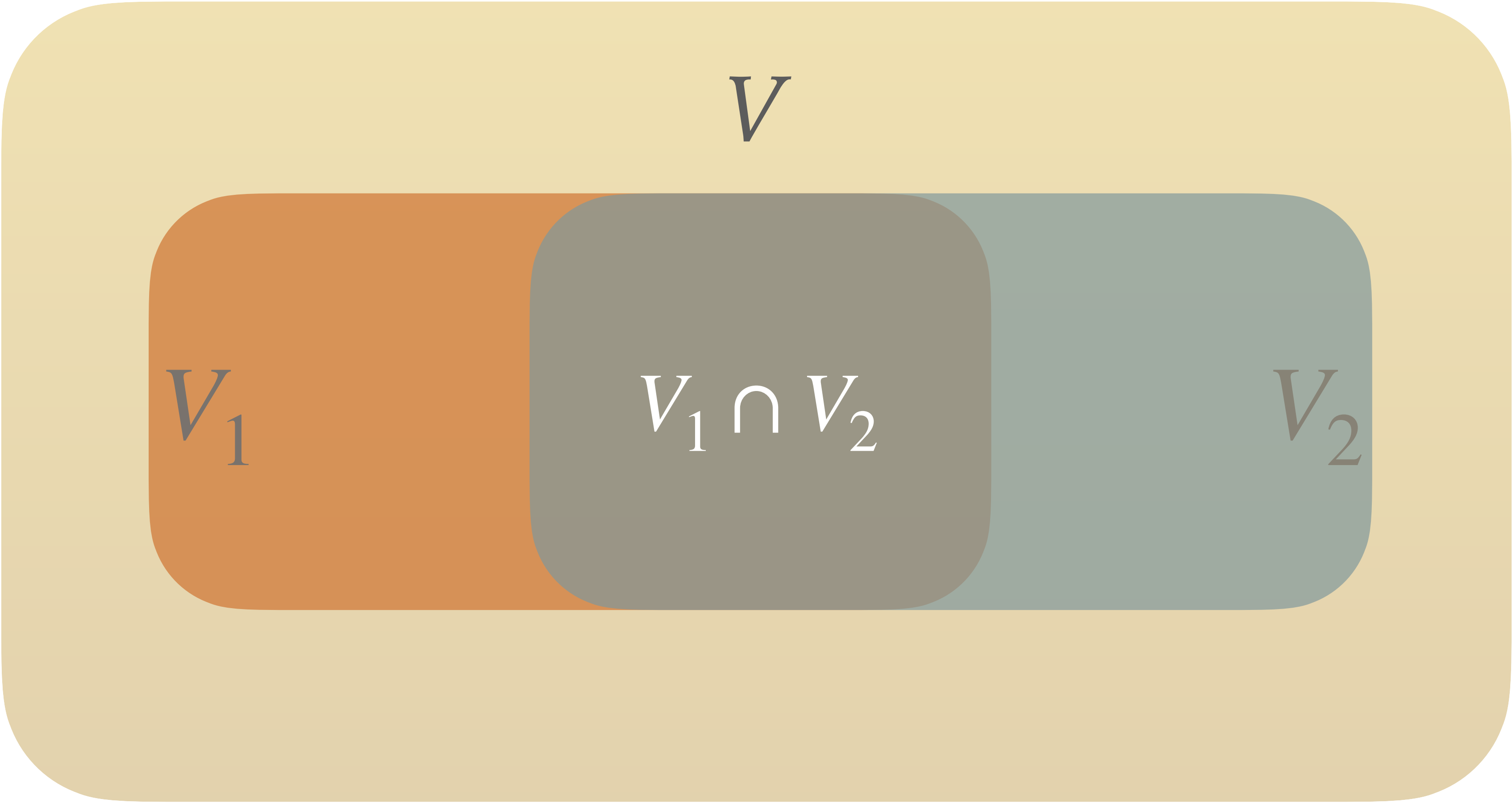

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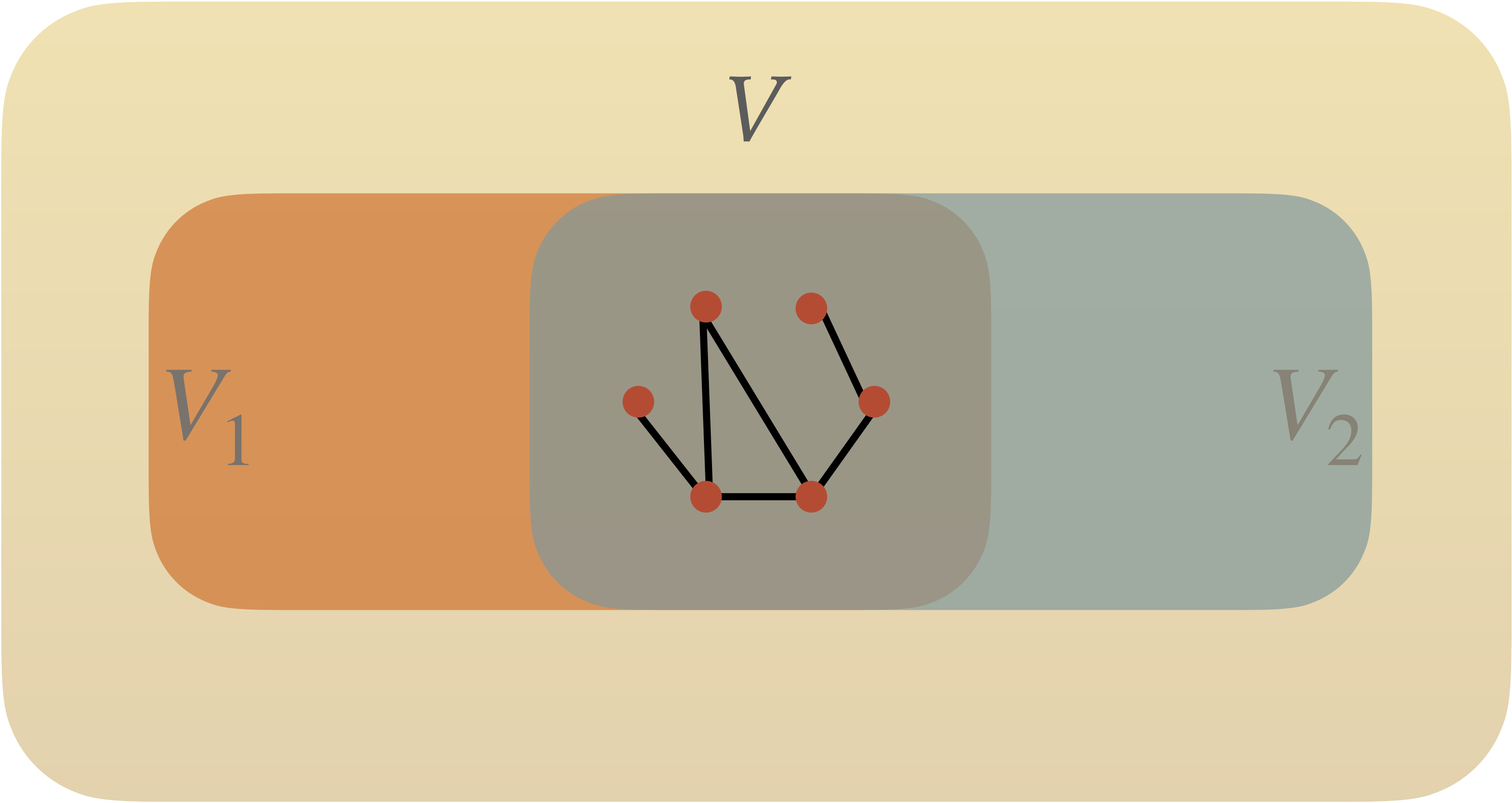

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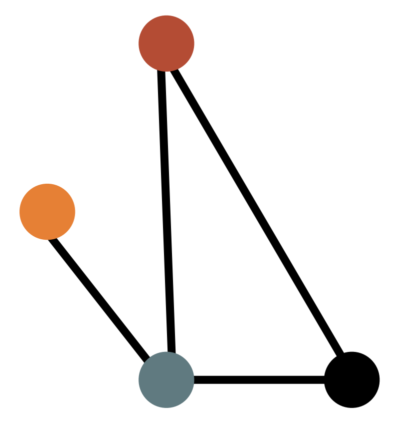

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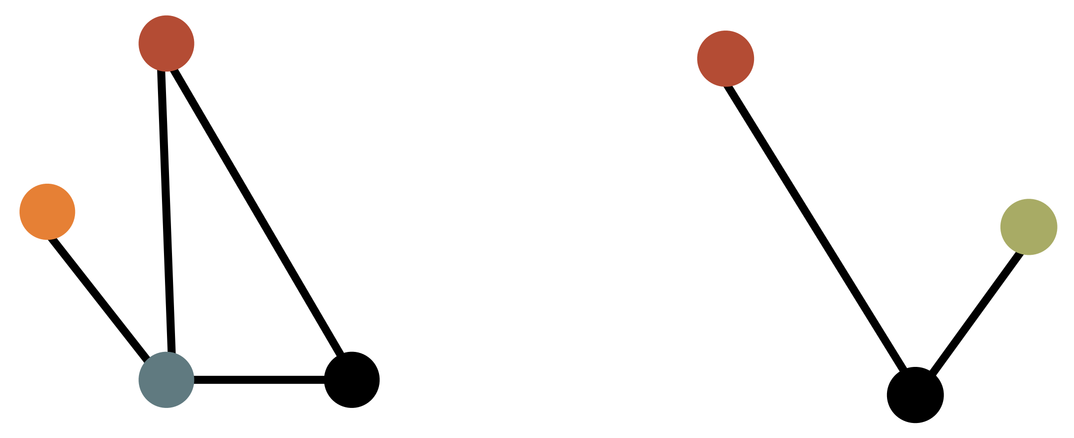

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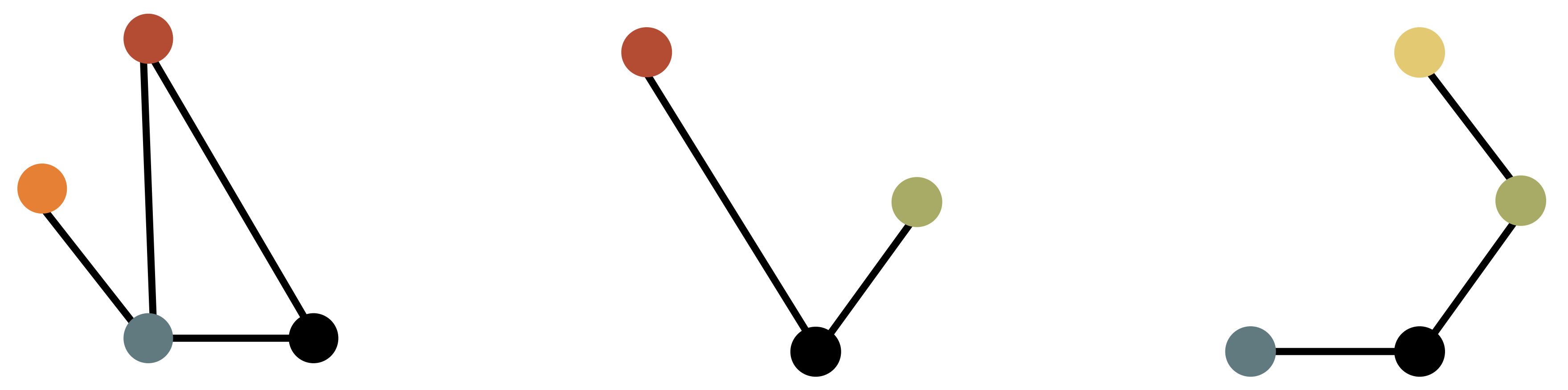

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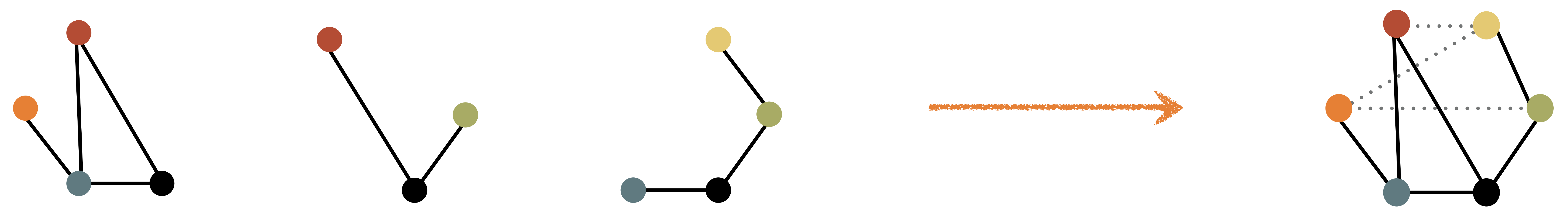
  So, if every vertex shows up in some trace, we will have consistently labeled all vertices, and if every pair of vertices shows up, we will have identified all the edges.



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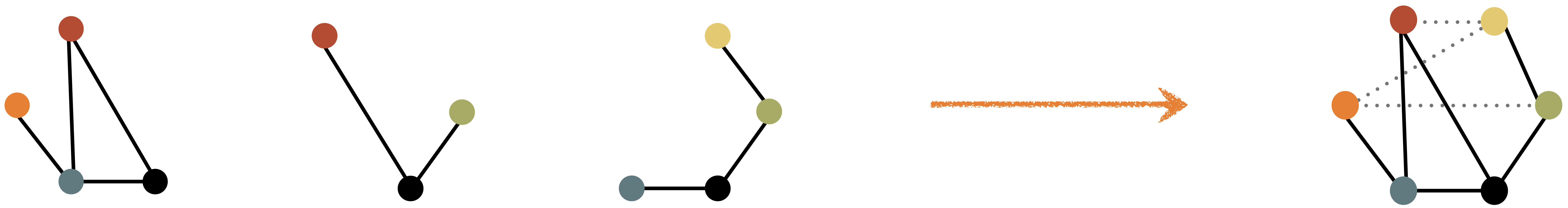

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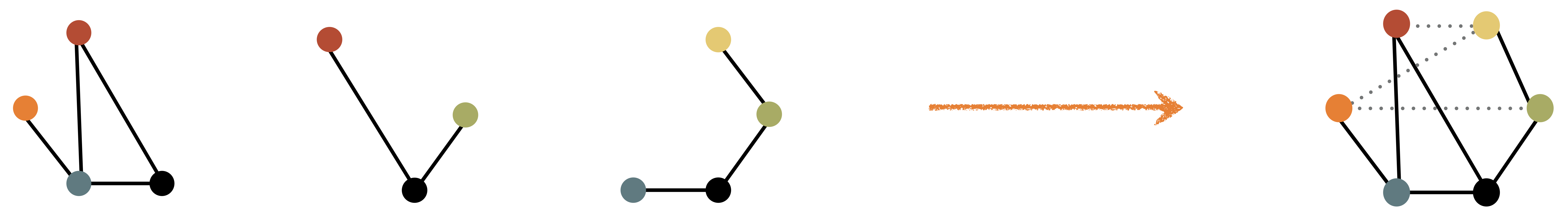

- Theorem (Main). For random graphs, $O(\log n)$ traces suffice for reconstruction.
- Extension. We can extend the result up to $q \approx 1 - \text{poly}(1/n)$ as well, using an extension to Müller, an extra subsampling step, and a modification to the proof.

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- Definition. The adjacency matrix of a graph $G = (V, E)$ on nodes $V = \{1, \ldots, n\}$ is an $n \times n$ binary matrix A with $A_{ij} = 1$ if and only if $(i, j) \in E$.

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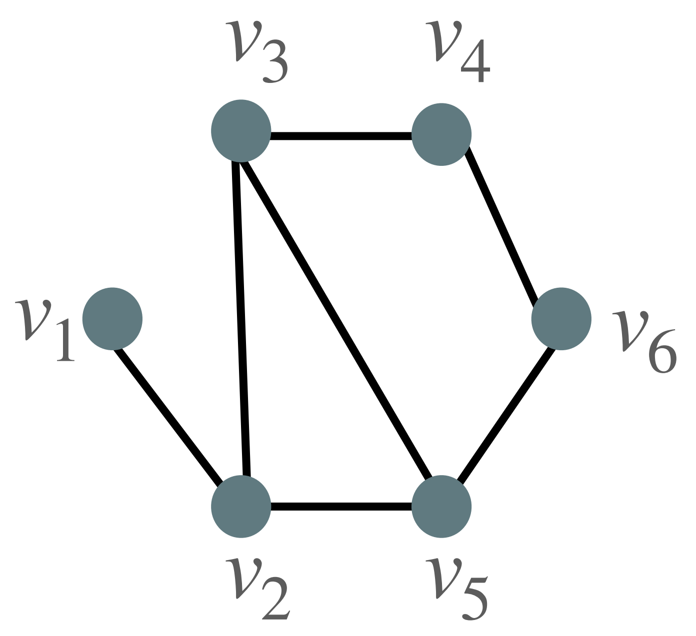

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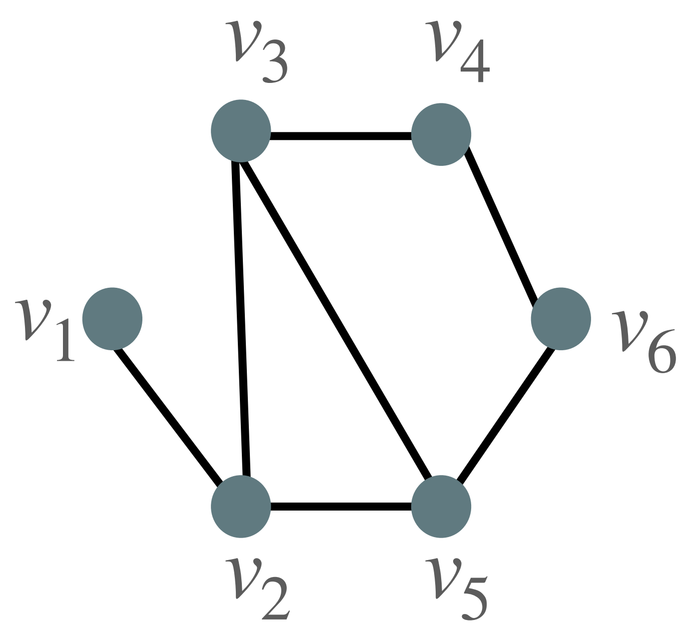

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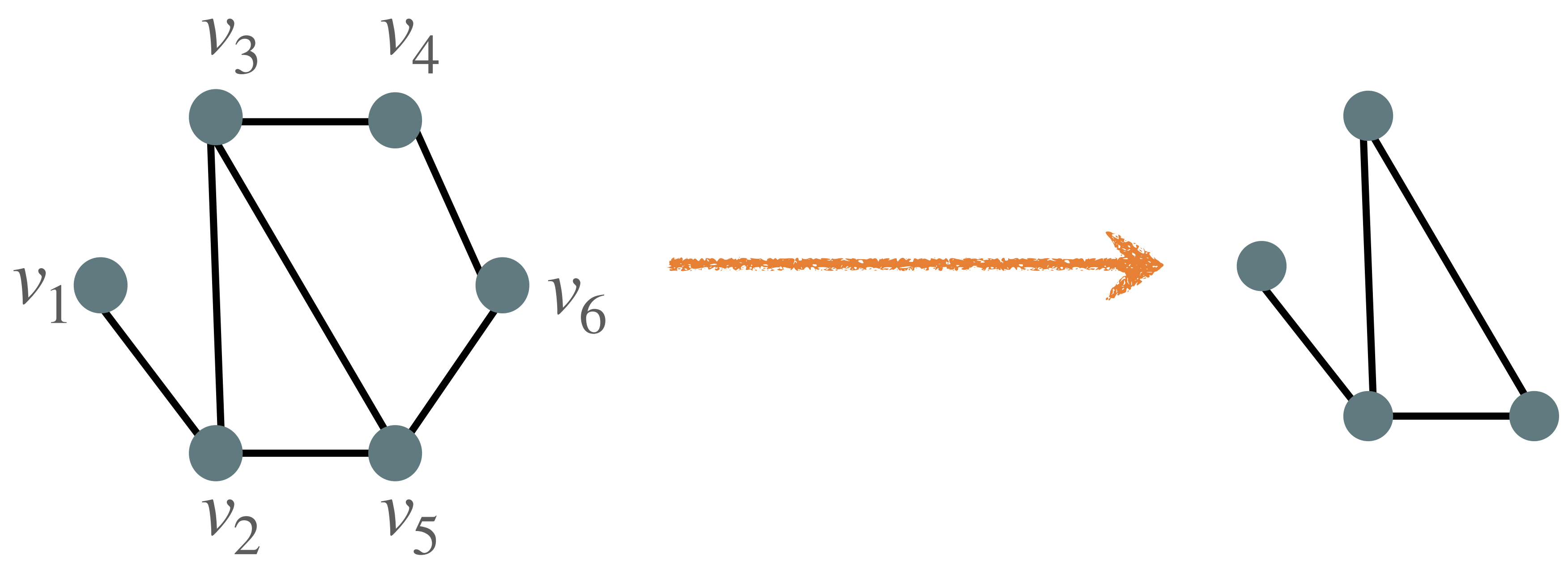

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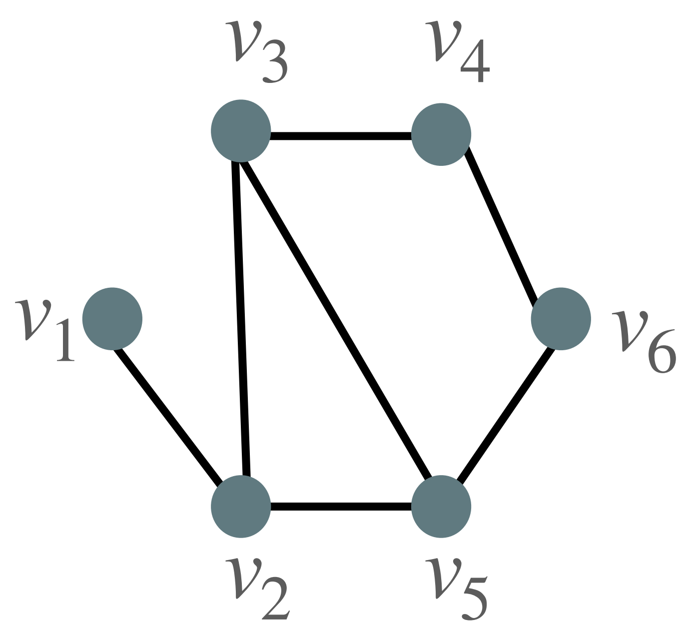

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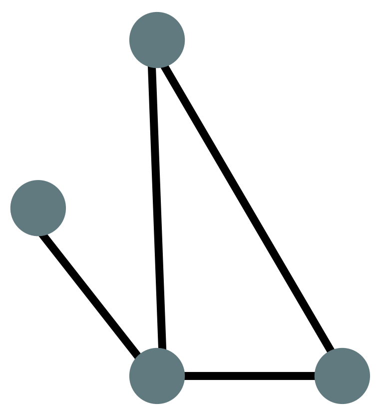

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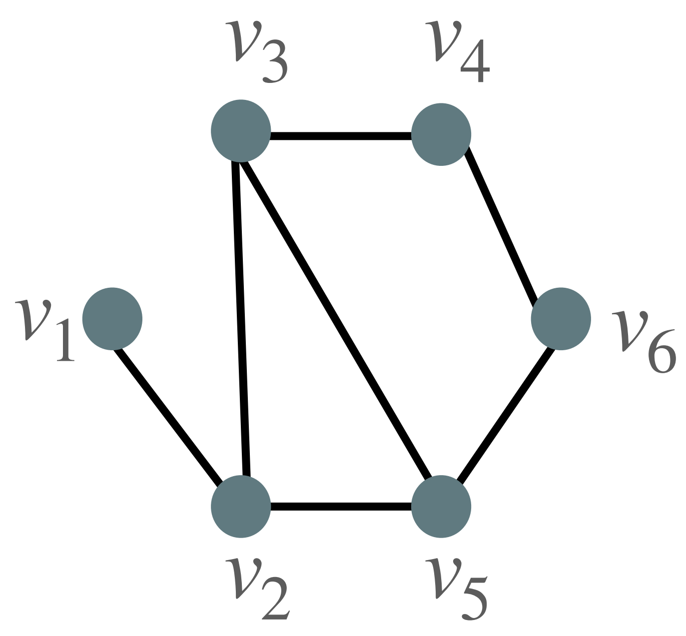

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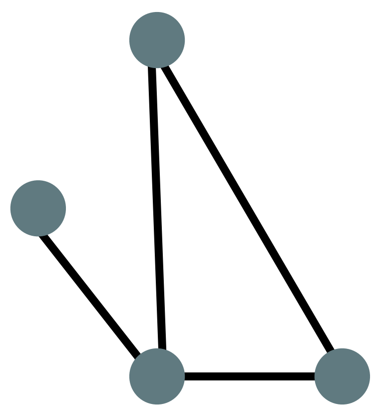

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- Question. How many random *symmetric* submatrices do we need to reconstruct the original graph?

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Can use complex analysis and moment estimation as in trace reconstruction, albeit with modifications.

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- Question. What are other structures that can have natural analogues for these questions, and what techniques can we inherit from these results?

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